

Microstrip Double-Periodic Grating of Continuous Curvilinear Metal Strips as a High-Impedance Surface^{*}

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ABSTRACT: Frequency dependence of phase reflection coefficient of a microstrip periodic grating of undulated strips is numerically investigated by means of the method of moments. The possibility of using of such structure as a high-impedance surface is shown. The influence of change in shape of the strips and in the parameters of dielectric substrate on the frequency characteristics of the grating is analyzed for the case on normal incidence of a plane electromagnetic wave. The possibility of electronic control of the grating characteristics is demonstrated.

INTRODUCTION

As is known, at normal reflection of a plane electromagnetic wave by perfectly conductive metal plane the vectors of the incident and reflected electric field strength have the same values but opposite directions on this plane owing to equality to zero of the strength of the total electric field \vec{E} in a perfect conductor. Therefore the reflection coefficient of such plane (which is called also “electric wall”) is equal to -1 , and the surface impedance, as follows from its definition $Z = E/H$, is equal to zero. If a perfectly conductive plane is replaced by a “magnetic wall” (which differs from electric one in that the tangential component of the strength of total magnetic field \vec{H} is equal to zero on its surface), the vectors of electric field strength of the incident and reflected wave will turn out the same on the reflecting surface. The reflection coefficient of a magnetic wall at normal incidence is equal to $+1$, and the surface impedance is infinitely large.

If a radiator is placed near an electric or magnetic wall, e.g. an electric dipole parallel to the wall, the radiation field is the sum of the “primary” field of the dipole (of its radiation field in free space) and the reflected field. The reflected field can be considered as the radiation field of the dipole image on the wall. The fundamental difference in the dipole images on the electric and magnetic wall

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consists in that they have opposite orientation. Let the distance between the dipole and the wall be small in comparison with the wavelength. Then the field reflected by the magnetic wall is added to the primary field of the dipole, instead of being canceled as in the case of a radiator near a metal surface. Therefore high-impedance surfaces are perspective in antenna engineering.

Any materials with properties of a perfect magnetic have not found in nature. Therefore they attempt to create artificial structures with typical properties of a magnetic wall if only for a narrow frequency range. For example, the properties of a grating of identical dipoles situated near a metal screen do not differ from reflective properties of an electric wall over the most region of microwave range. However the phase of reflection coefficient varies in a resonance way over a narrow frequency range depending on the resonance characteristics of the grating elements as a result of interaction between the grating and the metal surface above which the grating is situated. In that event the reflection coefficient is equal to +1 at resonance frequency. Such structure of dipoles situated near a conductive surface was examined in [1]. High-impedance surfaces in which wire elements have the shape of Hilbert's curve are represented in [2]. If narrow strips of complex shape are used as elements, their length per period can be much greater than the dimension of a cell of the periodic structure. Thus, it is possible to design microstrip reflective gratings (i.e. the gratings of thin conductive strips situated on dielectric layer which opposite side is screened) with a resonant wavelength much greater than their period. The frequency characteristics of reflection of some microstrip structures, in particular, of C-shaped elements, are considered in [3].

Plane microstrip grating of infinitely long continuous undular strips is an example of resonant high-impedance structure. Such grating is convenient for discrete variation of frequency characteristics by means of electronic control. For this purpose it is possible to make a discontinuity of a strip in each period and to include a control diode in it. A direct voltage impressed on a strip will transfer the diodes from a state with higher resistance (equivalently to that each strip of the grating consists of separate electrically disconnected elements) to a state with low resistance for a high-frequency current (the strip represents a continuous conductor).

In the case when continuous retuning of the characteristics is necessary, as a substrate we can choose a material which permittivity or permeability varies under the action of a direct voltage impressed on the adjacent strips of the grating or depending on the potential between the strips and the screen of the microstrip structure.

The purpose of the article is to study resonant properties of plane microstrip gratings of undulated strips and possibility to use these structures as artificial high-impedance surfaces.

PROBLEM STATEMENT AND METHOD OF NUMERICAL SOLUTION

Consider reflection of an electromagnetic wave at a periodic microstrip grating. The grating consists of identical rectangular cells with the sides d_x and d_y disposed, for example, as shown in Fig. 1. Plane periodic continuous perfectly conductive strips having an arbitrary shape in period and situated on the plane $z = 0$ are used as the grating elements. The shape of a strip element of the grating is assigned by the equation of its “center” line $\vec{\rho} = \vec{q}(s)$ (where $\vec{\rho} = \vec{e}_x x + \vec{e}_y y$) shown in Figs. 1(a)-1(c) by the dashed curve. It is supposed that the length S of the strip per period is much greater than its breadth. Width of the strip is defined in the direction of the normal to the center line and is equal to $2w$. The dielectric substrate has a depth h , relative permittivity ε and relative permeability μ . The plane $z = -h$ is a perfectly conductive screen. Let a plane electromagnetic wave be incident at such grating

$$\vec{E}^i = \vec{P} \exp(-i\vec{k}^i \vec{r}), \quad (1)$$

where \vec{P} is the polarization vector, $|\vec{P}| = 1$; \vec{k}^i is the wave vector of the incident wave, $k^i = k$, $k = \omega\sqrt{\varepsilon_0\mu_0}$. For simplicity we will consider only the case of inclined incidence under the condition of $k_x^i = 0$, i.e. $\vec{k}^i = \vec{e}_y k_y^i + \vec{e}_z k_z^i$. Here and below the time dependence is supposed to be of the $\exp(i\omega t)$ form.

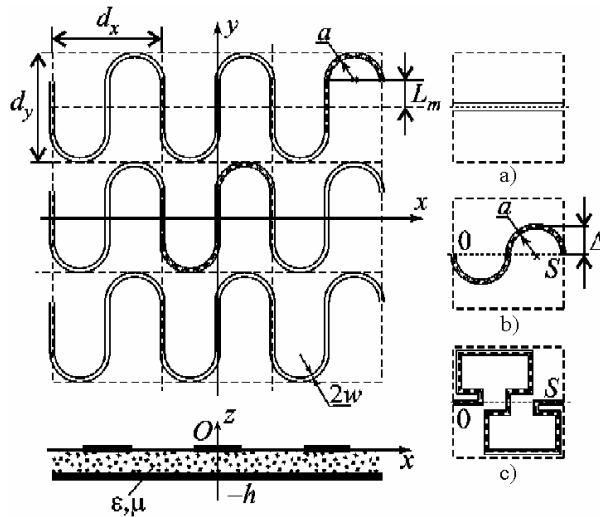


FIGURE 1. Plane bidirectional periodic grating and some variants of geometry of its period

We will find the field above the microstrip grating as a superposition of the field in absence of strip elements \vec{E}^d and the scattered field \vec{E}^s :

$$\vec{E} = \vec{E}^d + \vec{E}^s,$$

where $\vec{E}^d = \vec{E}^i + \vec{R} \exp(-i\vec{k}^s \vec{r})$, $\vec{k}^s = (0, k_y^i, -k_z^i)$ is the wave vector of the reflected plane wave. The vector \vec{R} is found from boundary conditions. The expression for \vec{R} in case of normal incidence is given in [3]. The existence of the scattered field conditioned the presence of the grating strips for which a current with some surface density $\vec{J}(\vec{\rho})$ is induced on the surface of the dielectric layer (in the plane $z = 0$).

If the surface current density is known, it is easy to obtain representation for the field all over space as the superposition of spatial harmonics:

$$\vec{E} = \vec{E}^d + \sum_{\zeta, \nu=-\infty}^{\infty} \bar{a}_{\zeta\nu} \exp\left\{-i\left[\vec{\kappa}_{\zeta\nu} \cdot \vec{\rho} + |z| \gamma_1(\vec{\kappa}_{\zeta\nu})\right]\right\},$$

where

$$\vec{\kappa}_{\zeta\nu} = \vec{e}_x 2\pi\zeta / d_x + \vec{e}_y (k_y^i + 2\pi\nu / d_y),$$

$$\gamma_1(\vec{\kappa}) = \sqrt{k^2 - |\vec{\kappa}|^2},$$

$$\bar{a}_{\zeta\nu} = \left\{ \tilde{\vec{E}}^s \tilde{\vec{J}} \right\}(\vec{\kappa}_{\zeta\nu}) / (d_x d_y),$$

$\left\{ \tilde{\vec{E}}^s \tilde{\vec{J}} \right\}(\vec{\kappa})$ is the operator defined in the following way [2]:

$$\tilde{\vec{E}}^s = \tilde{\vec{E}}_t + \vec{e}_z \tilde{\vec{E}}_z,$$

$$\left\{ \tilde{\vec{E}}_t \tilde{\vec{J}} \right\}(\vec{\kappa}) = \frac{k Z_0 \vec{\kappa}_\perp \left(\tilde{\vec{J}}, \vec{\kappa}_\perp \right)}{\kappa^2 \left[i(\gamma_2 / \mu) \cot \gamma_2 h - \gamma_1 \right]} + \frac{\gamma_1 \gamma_2 Z_0 \vec{\kappa} \left(\tilde{\vec{J}}, \vec{\kappa} \right)}{k \kappa^2 \left[i \varepsilon \gamma_1 \cot \gamma_1 h - \gamma_2 \right]},$$

$$\left\{ \tilde{\vec{E}}_z \tilde{\vec{J}} \right\}(\vec{\kappa}) = \frac{Z_0 \gamma_2 \left(\tilde{\vec{J}}, \vec{\kappa} \right)}{k \left[\gamma_2 - i \varepsilon \gamma_1 \cot \gamma_1 h \right]},$$

$$Z_0 = \sqrt{\mu_0 / \varepsilon_0} = 120\pi \text{ Ohm};$$

$$\vec{\kappa}_\perp = [\vec{\kappa}, \vec{e}_z], \quad \gamma_2(\vec{\kappa}) = \sqrt{k^2 \varepsilon \mu - |\vec{\kappa}|^2}, \quad \text{Im } \gamma_1 \leq 0, \quad \text{Im } \gamma_2 \leq 0.$$

Since the length of a strip per period is much greater than its width, we will take into account only the current component along the strip and neglect its component across one. Represent a longitudinal current as a series in piecewise linear basis functions, so-called “functions-covers”, with unknowns coefficients $\{c_\beta\}_{\beta=1}^N$. Their values to an accuracy of some constant correspond to values of the longitudinal current at the points equidistant at the value $\delta = S/N$ along the length S of a strip within the period. The distribution of longitudinal current density in cross direction is assigned by the function of the form of $1/\sqrt{1-(u/w)^2}$ (where u is the coordinate across the strip), which takes into account the feature of the longitudinal current density near edges.

Furthermore, from the solution algorithm detailed in [3,4], we will obtain the system of linear algebraic equations in the unknown amplitude factors of current $\{c_\beta\}_{\beta=1}^N$ by means of method of moments:

$$\sum_{\beta=1}^N A_{\alpha\beta} c_\beta = f_\alpha, \quad \alpha = 1, 2, \dots, N, \quad (2)$$

where

$$\begin{aligned} A_{\alpha\beta} = & \frac{\pi^2 \delta}{d_x d_y} w^2 \sum_{\zeta, \nu=-\infty}^{\infty} J_0(\vec{\kappa}_{\zeta\nu} \vec{n}_\alpha w) J_0(\vec{\kappa}_{\zeta\nu} \vec{n}_\beta w) \times \\ & \times \sin^2 c^2 \left(\frac{\delta}{2} \vec{\kappa}_{\zeta\nu} \vec{t}_\alpha \right) \sin^2 c^2 \left(\frac{\delta}{2} \vec{\kappa}_{\zeta\nu} \vec{t}_\beta \right) \times \\ & \times \left(\vec{t}_\alpha, \left\{ \vec{E}_i \vec{t}_\beta \right\}(\vec{\kappa}_{\zeta\nu}) \right) \exp \left[i \vec{\kappa}_{\zeta\nu} (\vec{q}_\beta - \vec{q}_\alpha) \right], \end{aligned} \quad (3)$$

$$f_\alpha = -\pi w \hat{t}_\alpha (\vec{P} + \vec{R}) J_0(\vec{k}^i \vec{n}_\alpha w) \exp(-i \vec{k}^i \vec{q}_\alpha),$$

$\text{sinc}(x) = \sin x/x$, $\vec{t}_\beta = \vec{t}(s_\beta)$, $\vec{n}_\beta = \vec{n}(s_\beta)$ are the unit vectors of the tangent and the normal to the center line of a strip, $\vec{q}_\beta = \vec{q}(s_\beta)$, $s_\beta = \beta\delta$, and $J_0(x)$ is the Bessel function of the first kind of a zero order.

ANALYSIS OF NUMERICAL RESULTS

For simplicity we will consider normal incidence of an electromagnetic wave (1) at a grating. If the both periods of the grating are less than the wavelength, $d_x/\lambda < 1$ and $d_y/\lambda < 1$, then there is only a fundamental spatial harmonic of reflected \vec{E}^r field, being propagated along the normal to the grating in the far-field zone:

$$\vec{E}^r = \vec{r} \exp(-ikz),$$

where the amplitude \vec{r} of the field reflected by the microstrip grating is determined by the expression $\vec{r} = \vec{R} + \vec{\alpha}_{00}$. For it to be able to determine the vector of amplitude of the reflected field by the assigned polarization vector of the incident field, it is convenient to introduce the reflection tensor into consideration

$$\widehat{R} = \begin{pmatrix} r_{xx} & 0 \\ 0 & r_{yy} \end{pmatrix}.$$

Its elements out of the principal diagonal are equal to zero, since the grating is symmetrical. If the reflection tensor is known, the vector of amplitude of the reflected field can be found according to the formula $\vec{r} = \widehat{R}\vec{P}$.

The values of reflection coefficients r_{xx} and r_{yy} depend on the shape of a strip and on the ratio between the dimensions of the strip per period and the wavelength. Taking into consideration that the reflection coefficients of the microstrip grating are complex quantities, the values of the surface impedance are determined by the expressions:

$$Z_x = \frac{E_x}{H_y} = Z_0 \left(\frac{1+r_{xx}}{1-r_{yy}} \right) = \frac{Z_0}{1-2\operatorname{Re} r_{xx} + |r_{xx}|^2} \left(1 - |r_{xx}|^2 + i2\operatorname{Im} r_{xx} \right),$$

$$Z_y = \frac{E_y}{H_x} = Z_0 \left(\frac{1+r_{yy}}{1-r_{xx}} \right) = \frac{Z_0}{1-2\operatorname{Re} r_{yy} + |r_{yy}|^2} \left(1 - |r_{yy}|^2 + i2\operatorname{Im} r_{yy} \right).$$

If in the substrate dielectric losses are not available, then the magnitudes of the reflection coefficients r_{xx} and r_{yy} are equal to unity, and consequently only the imaginary part of the surface impedance can be nonzero. For various values

of the normalized frequency d_y/λ the values of the real and imaginary parts of the reflection coefficients can be vary. This naturally causes changes in values of the surface impedance of the grating. Approximately it is possible to determine the wavelength in a microstrip line from the known formula [5]:

$$\lambda_e = \lambda / \sqrt{\varepsilon_e},$$

where ε_e is the effective permittivity

$$\varepsilon_e = \frac{\varepsilon + 1}{2} + \frac{\varepsilon - 1}{2\sqrt{1 + 5h/w}}.$$

The arguments of the complex coefficients r_{xx} and r_{yy} vary in a resonant way in the vicinity of the wavelengths divisible by the dimensions of a strip per period. If the incident wave E is polarized (the polarization vector is directed along the axis Ox), resonant wavelength is $\lambda_e \cong 2S$. The wavelength $\lambda_e \cong S$ becomes resonant for H -polarized wave, when the polarization vector is directed along the axis Oy .

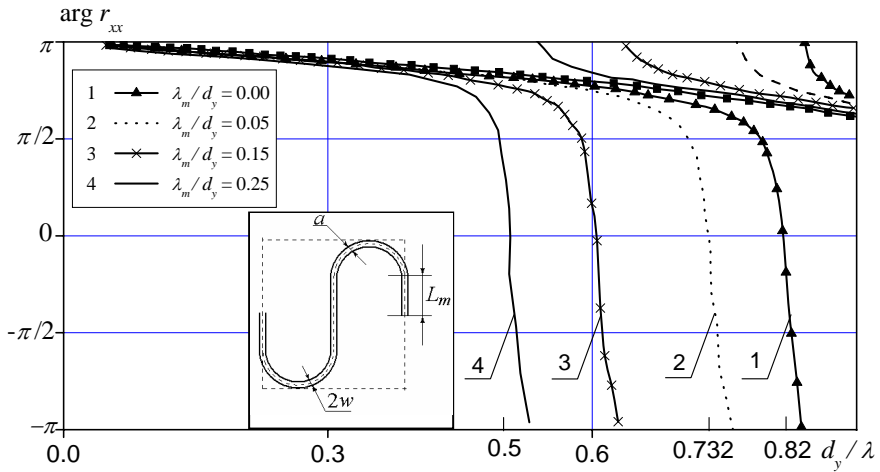


FIGURE 2. Frequency dependence of argument of the reflection coefficient of a grating of undular strips for various values of the amplitude of deviations from a straight line (E -polarization, $d_x = d_y$, $2w/d_y = 0.05$; $h_{sub}/d_y = 0.1$, $\varepsilon = 3$, $\mu = 1$, $\varepsilon_e = 2.22$, $a/d_y = 0.25$): curve 1 is $\lambda_m/d_y = 0$; curve 2 is $\lambda_m/d_y = 0.05$; curve 3 is $\lambda_m/d_y = 0.15$; curve 4 is $\lambda_m/d_y = 0.25$

Figure 2 represents the argument of the complex reflection coefficient depending on the normalized frequency for various values of the amplitude of deviations of the strip center line from a straight line at E -polarization. The described resonances are situated over the range $0.5 \leq d_y/\lambda \leq 0.82$ in the represented graph. The same dependence for H -polarization is shown in Fig. 3. In this case the resonances are situated over the range $0.24 \leq d_y/\lambda \leq 0.41$.

Figures 4 and 5 represents the frequency dependence of argument of the complex reflection coefficient of a grating of undular strips with maximum deviation from a straight line at various values of permittivity of the substrate and polarization of an incident wave. The normalized frequency of resonances decreases with permittivity of the substrate. The greater the corresponding value of ϵ_e , the greater is the quality factor of resonances. By quality factor, we understand characteristic of “half-width” of resonance, namely the ratio $f_r/\Delta f$, where f_r is the resonant frequency at which the argument of the complex reflection coefficient $\zeta = 0$, and $\Delta f = f_2 - f_1$, where f_1 and f_2 are the frequencies at which $\zeta = \pm\pi/2$. For example, at using of the values of the normalized frequency we obtain the quality factors $Q \approx 22$ and $Q \approx 12$ for E - and H -polarizations respectively at $\epsilon = 2$, $Q \approx 26$ and $Q \approx 14$ at $\epsilon = 3$, and $Q \approx 30$ and $Q \approx 16$ in the case of $\epsilon = 4$ from the presented graphs.

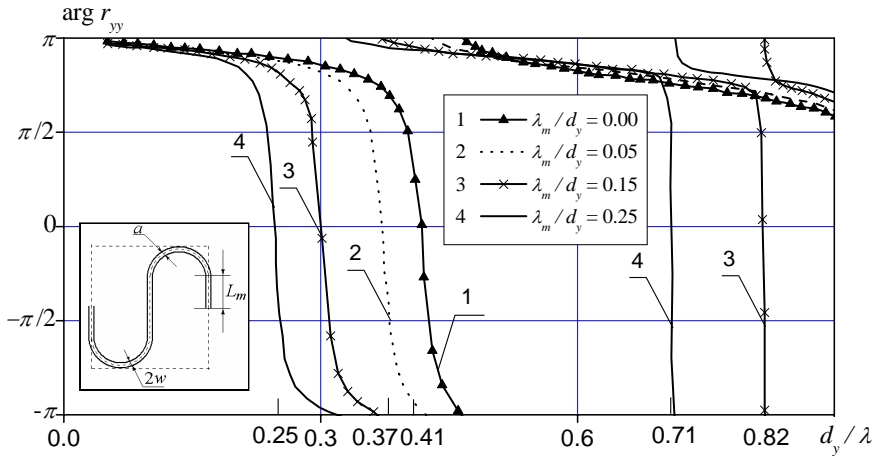


FIGURE 3. Frequency dependence of argument of the reflection coefficient of a grating of undular strips for various values of the amplitude of deviations from a straight line (H -polarization, $d_x = d_y$, $2w/d_y = 0.05$; $h_{sub}/d_y = 0.1$, $\epsilon = 3$, $\mu = 1$, $\epsilon_e = 2.22$, $a/d_y = 0.25$): curve 1 is $\lambda_m/d_y = 0$; curve 2 is $\lambda_m/d_y = 0.05$; curve 3 is $\lambda_m/d_y = 0.15$; curve 4 is $\lambda_m/d_y = 0.25$

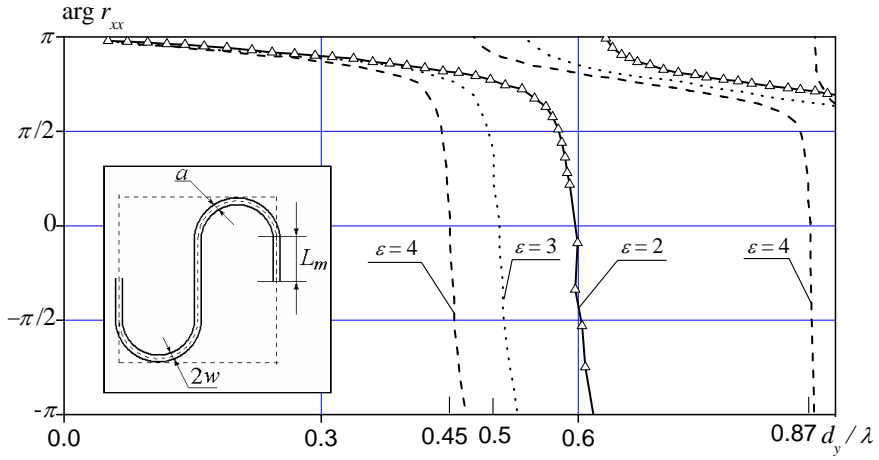


FIGURE 4. Frequency dependence of the argument of the reflection coefficient of a grating of undular strips at $L_m/d_y = 0.25$ at various values of ϵ of a substrate (E -polarization, $d_x = d_y$, $2w/d_y = 0.05$; $h_{sub}/d_y = 0.1$, $a/d_y = 0.25$, $\mu = 1$)

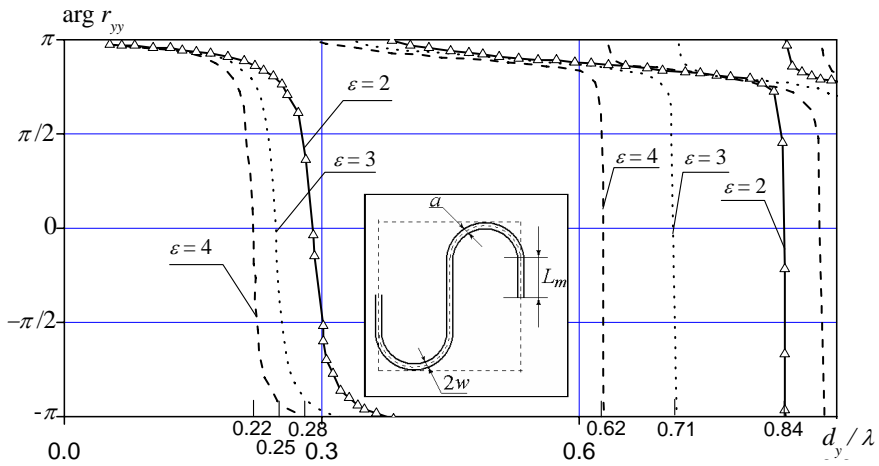


FIGURE 5. Frequency dependence of the argument of the reflection coefficient of a grating of undular strips at $L_m/d_y = 0.25$ at various values of ϵ of a substrate (H -polarization, $d_x = d_y$, $2w/d_y = 0.05$; $h_{sub}/d_y = 0.1$, $a/d_y = 0.25$, $\mu = 1$)

At shorter wavelengths λ_e resonances of higher order are to be exhibited. They correspond to $\lambda_e \cong 4S$ ($d_y/\lambda = 0.87$ at $\varepsilon = 4$, see Fig. 4) for a E -polarized incident wave and $\lambda \cong 3S$ ($0.62 \leq d_y/\lambda \leq 0.84$, see Fig. 5) for a H -polarized wave.

In summary it is necessary to note that a microstrip grating of continuous strips can have properties of a high-impedance surface also for smaller values of relative frequency d_y/λ than in the examples considered above. In this case the shape of a strip must be selected in such way to increase its length per period. One of great number of possible variants of grating strip configuration is shown in Fig. 1(c). It is obvious that increase in ε of the substrate also results in decrease in a resonant frequency.

CONCLUSION

The possibility of realization of properties of a high-impedance surface for plane microstrip gratings of continuous undulated strips in the neighborhood of their resonant frequencies is shown. The necessary frequency characteristics can be derived by selecting of the shape of grating strips and parameters of a dielectric substrate. Note that microstrip high-impedance structures can have very small thickness in comparison with a working wavelength. Due to this fact these structures can be successfully used. They are sufficiently easy to be made. The microstrip grating proposed is considerable promise as a high-impedance surface with characteristics controlled in electronic way, if it is used with control diodes included in discontinuities of strips.

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