Eur. Phys. J. Appl. Phys. **49**, 33005 (2010)
DOI: 10.1051/epjap/2010006

THE EUROPEAN PHYSICAL JOURNAL
APPLIED PHYSICS

Regular Article

Electromagnetic wave diffraction by array of complex-shaped metal elements placed on ferromagnetic substrate

S.L. Prosvirnin[1] and V.A. Dmitriev[2,a]

[1] Head of Theoretical Radio Physics Department, Institute of Radio Astronomy of National Academy of Sciences of Ukraine, 4 Chervonopraporna St., Kharkiv 61002, Ukraine
[2] Federal University of Para, P.O. Box 8619, Agencia UFPA, CEP 66075-900, Belem, Pará, Brazil

Received: 8 April 2009 / Accepted: 8 December 2009
Published online: 3 February 2010 – © EDP Sciences

Abstract. Full wave numerical study of electromagnetic wave reflection from and transmission through an array of thin, perfect conducting, planar strip complex-shaped elements placed on a magnetized ferrite substrate is carried out. Our results show, that the absorption level and the frequency band of the structure, when the ferromagnetic resonance and a metal element resonance are close in frequency, are larger than the corresponding characteristics of the ferrite layer without metal elements. A significant enhancement of the Faraday rotation has frequency shift with respect to the main resonance. Both right-handed and left-handed circular polarized incident waves interact effectively with a metal array placed on ferrite substrate unlike the known resonant interaction when only one kind (right- or left-handed) of circular polarized waves interacts with the ferrite slab without metal elements.

PACS. 42.25.Fx Diffraction and scattering – 78.20.Ls Magnetooptical effects – 41 Electromagnetism; electron and ion optics – 42 Optics

1 Introduction

Periodic arrays of planar complex-shaped metal elements placed on a dielectric substrate are used as frequency selective surfaces (FSS), polarizers, absorbers, etc. It is well known that ferrite materials in a magnetic biasing field are described by a permeability tensor and manifest resonance ferromagnetic response. Combining resonance responses of metal arrays and ferromagnetic characteristics of the ferrite substrate, one can expect to find new functional features of such structures.

An important peculiarity of the structures with a metal array placed on the ferrite substrate is the possibility of controlling their properties by a dc magnetic field. Significant change of transmission, reflection and absorption characteristics can be obtained as a result of such control.

It was shown in paper [1], that the frequency shift of resonance response of FSS with metal crosses on a ferrite substrate depends significantly on biasing magnetic field. But the presented in paper [1] results are concerned only with transmitted power, and the ferrite material of the substrate was considered to be lossless. Besides, the investigated frequency range was rather far from the ferromagnetic resonance where the possibilities of controlling are not very wide. Thus, the electromagnetic properties of such structures were not analyzed yet in detail.

The effects of interaction of electromagnetic waves with the suggested planar metamaterial can be described conveniently in terms of a scattering matrix. Symmetry of the problem which depends on the symmetry of the ferrite substrate, of the metal elements and of a dc magnetic field and its orientation, stipulates some restrictions on the operators entering the scattering matrix. Using the theory of magnetic groups we give three examples of such restrictions.

In our paper, we present also some results of numerical study of electromagnetic wave reflection from and transmission through an array of rosette-shaped metal elements placed on a magnetized ferrite substrate. The choice of complex-shaped elements gives us an opportunity to produce a symmetrical array having small sizes of elementary cell as compared with wavelength but nevertheless manifesting resonance properties.

2 Symmetry analysis

In Figure 1, we show some possible geometries of metamaterials composed of a ferrite substrate magnetized in the direction normal to it and of metal elements in the form of a rosette, case (a), and crosses, cases (b) and (c).

[a] e-mail: victor@ufpa.br

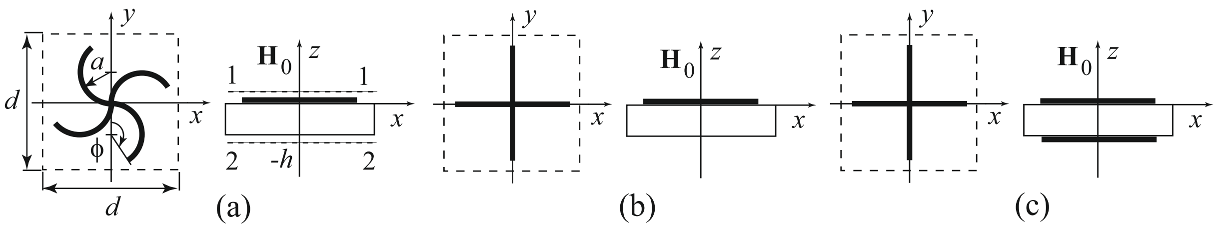

Fig. 1. Examples of FSSs placed on a ferromagnetic substrate: (a) one-sided rosette, (b) one-sided cross, (c) two-sided cross.

Table 1. Symmetry description of the metamaterials shown in Figure 1.

Characteristic	case (a)	case (b)	case (c)
Magnetic group of symmetry	$\mathbf{C_4}$	$C_{4v}(C_4)$	$D_{4h}(C_{4h})$
Number of group elements	4	8	16
Scattering matrix	$\begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ -S_{12} & S_{11} & -S_{14} & S_{13} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ -S_{32} & S_{31} & -S_{34} & S_{33} \end{pmatrix}$	$\begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ -S_{12} & S_{11} & -S_{14} & S_{13} \\ S_{13} & S_{14} & S_{33} & S_{34} \\ -S_{14} & S_{13} & -S_{34} & S_{33} \end{pmatrix}$	$\begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ -S_{12} & S_{11} & -S_{14} & S_{13} \\ S_{13} & S_{14} & S_{11} & S_{12} \\ -S_{14} & S_{13} & -S_{12} & S_{11} \end{pmatrix}$
Number of independent matrix elements	8	6	4

In case (b), the crosses are placed only on one side of the ferrite substrate, and they are on both sides of the substrate in case (c).

The resulting magnetic group of symmetry of the problems depends on the symmetry of the ferrite substrate, of the metallic elements and of a dc magnetic field and its orientation. This symmetry can be defined using Curie's principle of symmetry superposition [2]. The symmetry defines some restrictions on the elements (more exactly, operators because the reflection is not specular) of the scattering matrix.

The scattering matrix $\bar{\bar{S}}$ relates the tangential components of the electric fields of the scattered and incident waves in the planes 1–1 and 2–2 (see Fig. 1a) as follows: $(E^s_{x1}, E^s_{y1}, E^s_{x2}, E^s_{y2})^T = \bar{\bar{S}} \cdot (E^i_{x1}, E^i_{y1}, E^i_{x2}, E^i_{y2})^T$. Notice that the tangential components of an incident wave are defined by the angle of incidence. Using the theory of magnetic groups [2] we have determined the restrictions on the matrix elements. In Table 1, the magnetic groups of symmetry (in Schoenflies notations [2]) and the number of their elements, the calculated scattering matrices and the number of independent parameters of the matrices for the metamaterials in Figure 1 are presented.

Our method of calculation is based on the commutation relations for the scattering matrix $\bar{\bar{S}}$ and the 4×4 matrix representation $\bar{\bar{R}}$ of the symmetry elements (in fact, among the elements of the groups, we can use only the so-called generators). These commutation relations are $\bar{\bar{R}} \cdot \bar{\bar{S}} = \bar{\bar{S}} \cdot \bar{\bar{R}}$, $\bar{\bar{R}} \cdot \bar{\bar{S}} = \bar{\bar{S}}^T \cdot \bar{\bar{R}}$ for unitary and antiunitary elements of the corresponding magnetic group, respectively.

3 Derivation of analytical expression for the transfer matrix of a ferromagnetic layer

The fields, intensities, and polarization characteristics of the electromagnetic waves diffracted by the array of rosette-shaped elements were calculated by the full wave method described in [3]. Notice that earlier the method of [3] was applied to structures with nonmagnetic isotropic substrates. This approach is based on the method of moments for solution of vector integral equation for surface currents induced by the electromagnetic field on the array elements. The equation was derived with boundary conditions that assume a zero value for the tangential component of the electric field on metal strips. In our calculations, we used the Fourier transformations of fields and surface current distributions.

The main original part of our present work is the analytical expression obtained for the transfer matrix of normally magnetized ferromagnetic layer for the case of *arbitrary* orientation of wave vector of propagated or evanescent wave. This simplifies greatly the following numerical calculations.

In the following, we use common expressions for permittivity and permeability of *z*-axis biased ferrite [4] assuming ferrite material to be magnetically saturated and taking into account the magnetic losses

$$\varepsilon_f = \varepsilon_0 \varepsilon, \qquad \hat{\mu}_f = \mu_0 \begin{bmatrix} \mu & i\beta & 0 \\ -i\beta & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

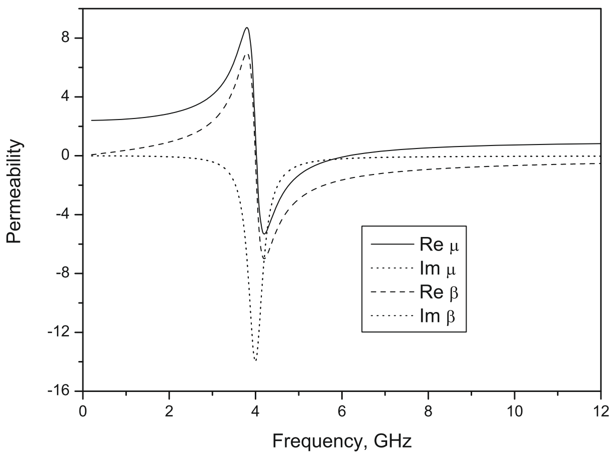

Fig. 2. Frequency dependencies of permeability parameters of z-axis biased ferrite substrate. Graphs of Imμ and Imβ are so close that they do not distinguish in the figure.

where

$$\mu = 1 + \chi' - i\chi'', \quad \beta = K' - iK'',$$

$$\chi' = \omega_0\omega_m[\omega_0^2 - \omega^2(1-\alpha^2)]D^{-1},$$

$$\chi'' = \omega\omega_m\alpha[\omega_0^2 + \omega^2(1+\alpha^2)]D^{-1},$$

$$K' = \omega\omega_m[\omega_0^2 - \omega^2(1+\alpha^2)]D^{-1}, \quad K'' = 2\omega^2\omega_0\omega_m\alpha D^{-1},$$

$$D = [\omega_0^2 - \omega^2(1+\alpha^2)]^2 + 4\omega_0^2\omega^2\alpha^2, \quad \omega_m = \mu_0\gamma M_s,$$

ε_0 and μ_0 are permittivity and permeability of free space, ω_0 is a ferromagnetic resonance frequency, α is a dimensionless damping constant, γ is a gyromagnetic ratio, M_s is a saturation magnetization. Typical ferrite parameters used in our calculations in the microwave region are the relative permittivity $\varepsilon = 10$, $\alpha = 0.05$, $\omega_0/2\pi = 4$ GHz, $\omega_m/2\pi = 5.6$ GHz. The value ω_m corresponds to saturation magnetization of 2000 G [4].

The frequency dependencies of the permeability parameters are presented in Figure 2. The values of Imμ and Imβ are so close to each other that their graphs coincide in the figure.

Solutions of the Maxwell equations

$$\nabla \times \mathcal{E}(\mathbf{r}) = -ikZ_0\hat{\mu}_f\mathcal{H}(\mathbf{r}), \quad \nabla \times \mathcal{H}(\mathbf{r}) = ikZ_0^{-1}\varepsilon_f\mathcal{E}(\mathbf{r}) \tag{2}$$

can be sought in the form

$$\mathcal{E}(\mathbf{r}) = e^{-ik_x x - ik_y y}\mathbf{E}(z), \quad \mathcal{H}(\mathbf{r}) = e^{-ik_x x - ik_y y}\mathbf{H}(z), \tag{3}$$

where $k = \omega\sqrt{\varepsilon_0\mu_0}$, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.

It is easy to derive the equation

$$\frac{d}{dz}\bar{F}(z) = \bar{\bar{M}} \cdot \bar{F}(z) \tag{4}$$

for the 4-vector $\bar{F}(z) = (E_x, E_y, H_x, H_y)^T$ which presents the tangential components of the fields, by using equations (2) and expressions (3). Elements of the matrix $\bar{\bar{M}}$ of the equation (4) are the following:

$$m_{11} = m_{12} = m_{21} = m_{22} = m_{33} = m_{34} = m_{43} = m_{44} = 0,$$

$$m_{13} = -\frac{Z_0}{k\varepsilon}(k^2\varepsilon b + ik_xk_y), \quad m_{14} = -\frac{iZ_0}{k\varepsilon}(k^2\varepsilon a - k_x^2),$$

$$m_{23} = \frac{iZ_0}{k\varepsilon}(k^2\varepsilon a - k_y^2), \quad m_{34} = -\frac{Z_0}{k\varepsilon}(k^2\varepsilon b - ik_xk_y),$$

$$m_{31} = \frac{ik_xk_y}{kZ_0c}, \quad m_{32} = \frac{i}{kZ_0c}(k^2\varepsilon c - k_x^2),$$

$$m_{41} = -\frac{i}{kZ_0c}(k^2\varepsilon c - k_y^2), \quad m_{42} = -\frac{ik_xk_y}{kZ_0c}.$$

Next, we assume that the vector $\bar{F}(z)$ is known in the plane $z = z_0$ and look for an analytical solution of the Cauchy problem for equation (4) in the form

$$\bar{F}(z) = \bar{\bar{T}}(z, z_0) \cdot \bar{F}(z_0). \tag{5}$$

The 4×4-matrix $\bar{\bar{T}}(z, z_0)$ is a transfer matrix for the ferrite homogeneous layer magnetized in the direction normal to it. The matrix elements depend on the parameters k_x and k_y which can take any real values and define the direction of propagation of spatial partial waves both for nonevanescent and evanescent ones. The analytic expressions of the matrix elements are unwieldy formulas derived by using a computer software for analytic transformations. The small paper volume does not allow us present here these general expressions. Therefore let us present elements of the matrix $\bar{\bar{T}}(z, z_0)$ for only simple special case $k_x = k_y = 0$. They are the following:

$$t_{11} = t_{22} = t_{33} = t_{44} = \frac{1}{2}(\cos g_+z + \cos g_-z),$$

$$t_{12} = -t_{21} = t_{34} = -t_{43} = \frac{i}{2}(\cos g_+z - \cos g_-z),$$

$$t_{13} = t_{24} = -\frac{Z_0}{2\sqrt{\varepsilon}}(\sqrt{\mu+\beta}\sin g_+z - \sqrt{\mu-\beta}\sin g_-z),$$

$$t_{14} = -t_{23} = -\frac{iZ_0}{2\sqrt{\varepsilon}}(\sqrt{\mu+\beta}\sin g_+z + \sqrt{\mu-\beta}\sin g_-z),$$

$$t_{31} = t_{42} = \frac{\sqrt{\varepsilon}}{2Z_0}\left(\frac{\sin g_+z}{\sqrt{\mu+\beta}} - \frac{\sin g_-z}{\sqrt{\mu-\beta}}\right),$$

$$t_{32} = -t_{41} = \frac{i\sqrt{\varepsilon}}{2Z_0}\left(\frac{\sin g_+z}{\sqrt{\mu+\beta}} + \frac{\sin g_-z}{\sqrt{\mu-\beta}}\right),$$

where $g_\pm = k\sqrt{\varepsilon}\sqrt{\mu \pm \beta}$.

We used the expressions of matrix elements of the general transfer matrix to derive an integral equation for the density of a surface current along the metal elements of the array.

4 Analysis of numerical results

Below, we present some numerical results for the array of rosettes shown in Figure 1a. The width of metal strips of elements is assumed to be narrow as compared with

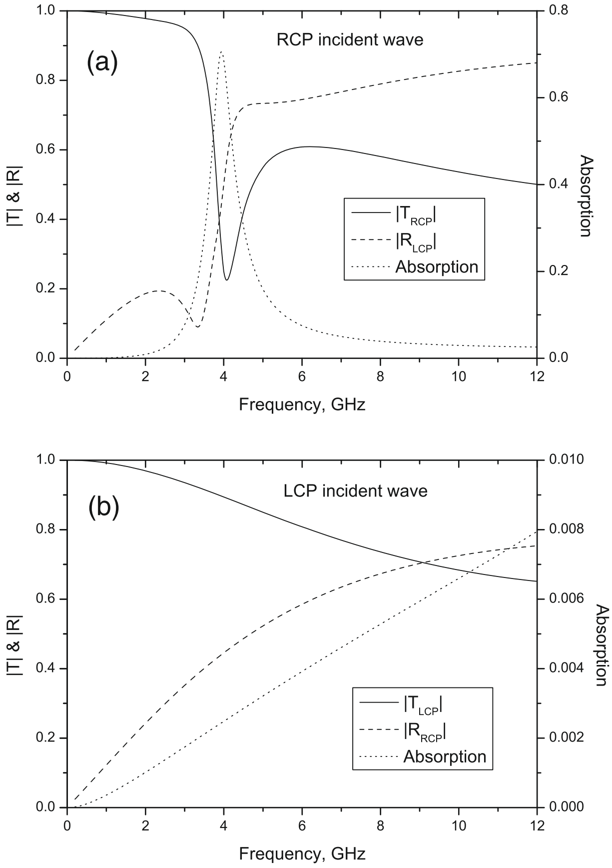

Fig. 3. Frequency dependencies of transmission and reflection coefficients defined by field amplitudes and relative absorption power A of magnetized ferrite slab for (a) RCP and (b) LCP normally incident waves.

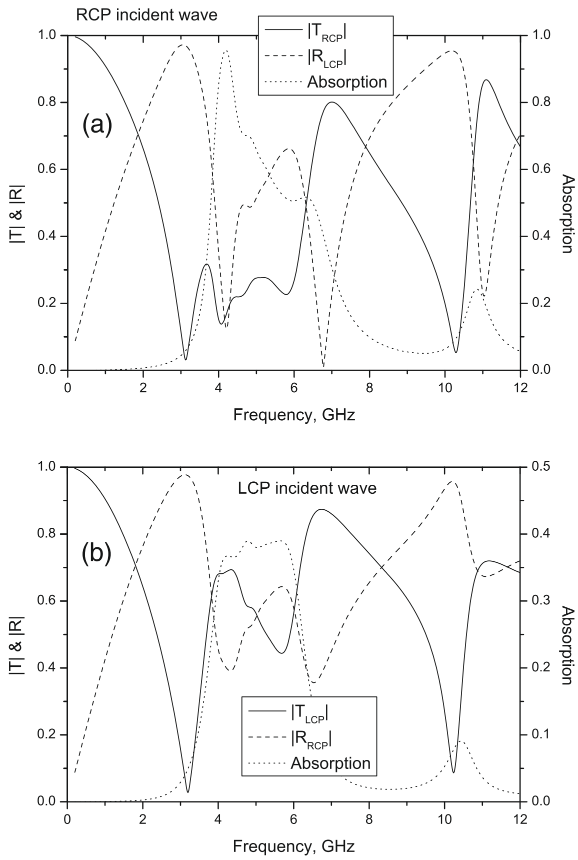

Fig. 4. Frequency dependencies of transmission and reflection coefficients defined by field amplitudes and relative absorption power A for the array of rosette-shaped elements placed on magnetized ferrite slab for (a) RCP and (b) LCP normally incident wave.

their stretched length and with the wavelength. The sizes of the array elements are chosen in such a way that their first low-frequency resonances to be near the ferromagnetic resonance frequency. The rosettes are arranged in square cell array with the period $d = 15$ mm. The width of the infinitely thin perfectly conducting strips is assumed to be 0.8 mm, the radius $a = 5$ mm, and the bending angle $\phi = 120°$. The magnetized ferrite substrate with above mentioned relative permittivity and permeability parameters has the thickness $h = 1.5$ mm.

We present here a comparison of transmission, reflection, absorption, and polarization properties of the array placed on the magnetized ferrite substrate and of the ferrite slab without array. It is assumed below that a plane electromagnetic wave is normally incident from the space region $z > 0$.

As it is well known, the eigenwaves of unbounded ferrite medium with longitudinal magnetization are right-handed and left-handed circular polarized (RCP and LCP) waves. Thus normally incident RCP or LCP wave does not change its polarization when transmitted through ferrite slab or through ferrite with an infinitely thin array with 4-fold rotational symmetry. But the polarization of RCP or LCP wave changes to opposite at the reflection.

In Figures 3 and 4 we present frequency dependencies of absolute values of the transmission T and reflection R coefficients defined by field amplitudes and also the relative absorption defined as $A = 1 - |T|^2 - |R|^2$ for the single ferrite slab and for the ferrite slab with the array structure in the case of circular polarized incident waves. An intensive absorption around the ferromagnetic resonance may be observed only for RCP wave incidence on the slab and for both RCP and LCP wave incidence on the array. Notice that we use here the optical definition of right-handed and left-handed circular polarization, i.e. electric field rotates respectively clockwise and anticlockwise for observation opposite to the wave propagation direction.

There are resonances of transmission and reflection observed in the same frequencies for the RCP and LCP incident wave due to resonance properties of elements of array. A level of the array absorption and the width of frequency band of large absorption are approximately the same for both kinds of circular polarization of the incident wave (see Fig. 4). The radical difference of the slab and the array properties may be seen for the LCP incident wave. Ferromagnetic resonance of the slab does not appear for the LCP wave. An explanation for the significant absorption of the ferrite layer with array is excitation of a complex electromagnetic field of evanescent waves inside the substrate.

A polarization state of the electromagnetic waves transmitted through and reflected from the ferromagnetic slab or the array structure may be characterized by angle of azimuth and ellipticity (see Fig. 5). The polarization azimuth θ and the ellipticity angle η are defined from the field amplitude using the standard definitions $\tan 2\theta = s_2/s_1$, $\sin 2\eta = s_3/s_0$, where s_i are the Stokes parameters calculated from the components of the electric field in the right-hand orthogonal frame.

We present in Figure 5 frequency dependencies for rotation and ellipticity of electromagnetic field transmitted through and reflected from the ferrite slab and the array placed on the ferrite substrate. It is assumed that normally incident wave has linear polarization along x-axis. We found an enhancement of the rotation of electric field and change the sign of rotation in the vicinity of the resonances of the array metal elements. The observed significant enhancement of the Faraday rotation has frequency shift with respect to the ferromagnetic resonance.

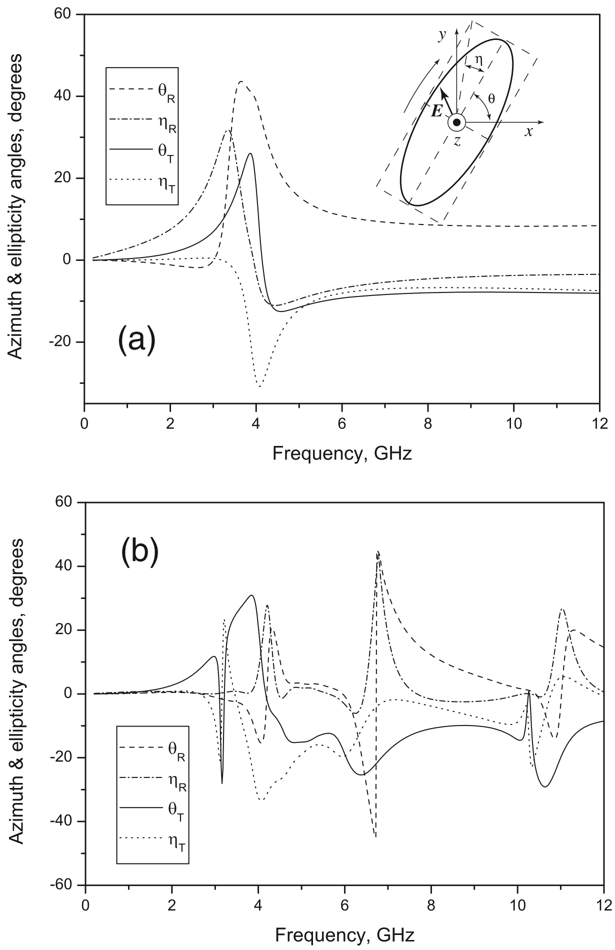

Fig. 5. Frequency dependencies of azimuth and ellipticity angles of electromagnetic waves transmitted through and reflected from (a) the magnetized ferrite slab and (b) the array placed on magnetized ferrite substrate. A normally incident wave has linear x-axis polarized electric field. Definition of polarization angles is shown in the insert of (a). In the definition a wave propagation direction is assumed coincide with z-axis in the insert. The polarization parameter θ is the azimuth of polarization, and η is the ellipticity angle. The lower indexes R and T of azimuth and ellipticity angles indicate polarization parameters of reflected and transmitted waves correspondingly. The positive and negative sign of η corresponds to right-handed polarization (i.e. **E** rotates clockwise as it is shown by arrow) and left-handed polarization for observation opposite to the wave propagation direction.

5 Conclusion

Our results show that the absorption level and the frequency band of the discussed periodic structure of rosette-shaped elements, when the ferromagnetic resonance and a metal element resonance approximately coincide, are larger than the corresponding parameters of the ferrite substrate without metal elements. We also observe a significant enhancement of the Faraday rotation. The enhancement of rotation has frequency shift with respect to the ferromagnetic resonance. This effect appears when the ferromagnetic resonance is close to the metal element resonance and also when these resonances are far from each other. The sign of rotation changes with a small variation of frequency in the vicinity of resonance. Both RCP and LCP incident waves interact with the array placed on ferrite substrate in contrast to the case of ferrite slab without metal elements.

The authors would like to acknowledge the financial support of the Brazilian agency CNPq, and National Academy of Sciences of Ukraine (Grant No. 1-02-a).

References

1. G.Y. Li, Y.C. Chan, T.S. Mok, J.C. Vardaxoglou, Int. J. Electron. **78**, 1159 (1995)
2. A.A. Barybin, V.A. Dmitriev, *Modern Electrodynamics and Coupled-Mode Theory: Application to Guided-Wave Optics* (Rinton Press, Princeton, NJ, 2002)
3. S.L. Prosvirnin, J. Commun. Technol. Electron. **44**, 635 (1999)
4. R.E. Collin, *Foundation for Microwave Engineering*, 2nd edn. (Wiley-Interscience, New York, 2001)